

1) Suppose  $(0, 2)$  is a critical point of a function  $f$  with continuous second derivatives. In each case, what can you say about  $f$ ?

a)  $f_{xx}(0, 2) = -1$ ,  $f_{xy}(0, 2) = 6$ ,  $f_{yy}(0, 2) = 1$

b)  $f_{xx}(0, 2) = -1$ ,  $f_{xy}(0, 2) = 2$ ,  $f_{yy}(0, 2) = -8$

c)  $f_{xx}(0, 2) = 4$ ,  $f_{xy}(0, 2) = 6$ ,  $f_{yy}(0, 2) = 9$

2) Find the local maximum and minimum values and saddle point(s) of the function.

a)  $f(x, y) = 9 - 2x + 4y - x^2 - 4y^2$

b)  $f(x, y) = x^4 + y^4 - 4xy + 2$

c)  $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$

d)  $f(x, y) = x^2 + y^2 + \frac{1}{x^2 y^2}$

- 3) Find the critical points of the function  $f(x, y, z) = x^2 + (y - 3)^2 + (z + 1)^2$  from the form of the function determine whether a relative maximum or a relative minimum occurs at each point.
- 4) Find the absolute maximum and minimum values of  $f$  on the set  $D$ .
- $f(x, y) = 1 + 4x - 5y$ ,  $D$  is the closed triangular region with vertices  $(0, 0)$ ,  $(2, 0)$ ,  $(0, 3)$ .
  - $f(x, y) = 4x + 6y - x^2 - y^2$ ,  $D = \{(x, y) \mid 0 \leq x \leq 4, 0 \leq y \leq 5\}$
  - $f(x, y) = xy^2$ ,  $D = \{(x, y) \mid x \geq 0, y \geq 0, x^2 + y^2 \leq 3\}$

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- 5) Find the shortest distance from the point  $(2, 1, -1)$  to the plane  $x + y - z = 1$ .
- 6) Find three positive numbers whose sum is 100 and whose product is a maximum.
- 7) Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane  $x + 2y + 3z = 6$ .
- 8) Find the dimensions of the rectangular box with largest volume if the total surface area is given as  $64 \text{ cm}^2$ .